This supplementary text outlines the framework of mathematical models, the flowchart of COVID-19 infection, equilibrium solutions of the transmission dynamics, reproduction number, the correlation between the reproduction number and vaccination rates. We herein include the simulation outcomes for severe cases corresponding to the major outcomes of the manuscript. The definitions and values of parameters utilized in the simulations are illustrated subsequently. The supplementary material mainly consists of three sections as follows.

Supplementary Methods

Suppose a susceptible-vaccinated-infected-recovered-susceptible[SVIRS] model and a population of size N, in which each individual can be vaccinated or unvaccinated (Fig. S1). Assume the birth rate of the population is equivalent to the death rate, individuals recover from infection at the rate and the thereafter immunity wanes at rate after recovery. Full susceptible individuals get primary doses 1&2 vaccinated at rate and , and get the booster dose 3 vaccinated at rate respectively. The immunity protection of doses 1&2&3 declines at rate , and respectively. Vaccinated individuals whose immunity wanes are infected at rate , and . , and denote the administration initiation timing of doses 1, 2, and 3. is the infection rate after vaccination of dose . Partially susceptible individuals can be vaccinated again and obtain the immunity protection equivalent to doses 1&2&3 at rate , and . is the likelihood of being severe disease due to the infection of COVID-19.



Figure S1: COVID-19 transmission dynamics
Dose 1 vaccination
Dose-1 immunity wanes
Dose-2 immunity wanes
Primary infection

- Infection after vaccination Recovered
- Partial susceptibility
 Secondary infection
- Dose 3 vaccination Dose-3 immunity wanes
- Infection after dose-3 immunity wanes Full susceptibility
- Infection after dose-2 immunity wanes

The model governing the epidemic transmission dynamics can be expressed as follows:

$$\frac{dS_{P}}{dt} = \mu \cdot \beta S_{P} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{I} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{1}}] - (s_{vas} v + \mu) S_{P} \quad (1)$$

$$\frac{dI_{P}}{dt} = \beta S_{P} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{I} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{3}}] - (v + \mu) I_{P} \quad (2)$$

$$\frac{dR}{dt} = \gamma [I_{P} + I_{S} + I_{V} + I_{S_{1}} + I_{S_{2}} + I_{S_{2}}] - (\delta + \mu) R \quad (3)$$

$$\frac{dS_{2}}{dt} = \delta R \cdot \epsilon \beta S_{S} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{1}}] - (v + \mu) I_{S} \quad (4)$$

$$\frac{dI_{S}}{dt} = \epsilon \beta S_{S} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{1}}] - (v + \mu) I_{S} \quad (5)$$

$$\frac{dV_{1}}{dt} = s_{vav} S_{P} + cs_{vav} S_{S} \cdot \varepsilon_{v_{1}} \beta V_{1} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - (v + \mu) V_{1} \quad (6)$$

$$\frac{dV_{2}}{dt} = ds_{vav} V_{S} + cs_{vav} V_{S} \cdot \varepsilon_{v_{1}} \beta V_{2} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - (v + \mu) V_{1} \quad (6)$$

$$\frac{dV_{2}}{dt} = ds_{vav} V_{S} + cs_{vav} V_{S} \cdot \varepsilon_{v_{1}} \beta V_{2} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - (v + \mu) V_{1} \quad (6)$$

$$\frac{dV_{2}}{dt} = ds_{vav} V_{S} + cs_{vav} V_{S} \cdot \varepsilon_{v_{1}} \beta V_{2} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - (v + \mu) V_{1} \quad (6)$$

$$\frac{dV_{2}}{dt} = (1 - c - d) s_{vav} V_{S} + \omega_{V} S_{vav} V_{1} - \varepsilon_{v_{1}} \beta V_{S} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - (v + \mu) I_{V} \quad (9)$$

$$\frac{dS_{s_{1}}}{dt} = \rho_{1} V_{1} - \varepsilon_{1} \beta S_{s_{1}} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - \mu S_{S_{1}} \quad (10)$$

$$\frac{dS_{s_{1}}}{dt} = \rho_{2} V_{2} - \varepsilon_{2} \beta S_{s_{1}} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] - \mu S_{S_{2}} \quad (11)$$

$$\frac{dS_{s_{1}}}{dt} = \rho_{2} V_{2} - \varepsilon_{2} \beta S_{s_{2}} [I_{P} + \alpha I_{S} + \alpha_{V} I_{V} + \alpha_{1} I_{S_{1}} + \alpha_{2} I_{S_{2}} + \alpha_{3} I_{S_{2}}] -$$

$$I_{SR} = \lambda_P I_P + \lambda_S I_S + \lambda_V I_V + \lambda_1 I_{S_1} + \lambda_2 I_{S_2} + \lambda_3 I_{S_3}$$
(16)
$$S_{vax} = \begin{cases} 0, & t < t_{vax} \\ 1, & t \ge t_{vax} \end{cases}$$
(17)

$$S_{vaxl} = \begin{cases} 0, & t < t_{vaxl} \\ 1, & t \ge t_{vaxl} \end{cases}$$
(18)

$$S_{vax2} = \begin{cases} 0, & t < t_{vax2} \\ 1, & t \ge t_{vax2} \end{cases}$$
(19)

In a disease-free equilibrium, no incidence of infections occurs and thus the system can be rephrased to below:

$$\frac{dS_{p}}{dt} = \mu \cdot (\nu + \mu)S_{p} = 0 \quad (20)$$

$$\frac{dV_{1}}{dt} = \nu S_{p} - (\omega + \rho_{1} + \mu)V_{1} = 0 \quad (21)$$

$$\frac{dV_{2}}{dt} = \omega V_{1} - (\omega_{1} + \rho_{2} + \mu)V_{2} = 0 \quad (22)$$

$$\frac{dV_{3}}{dt} = \omega_{1}V_{2} - (\rho_{3} + \mu)V_{3} = 0 \quad (23)$$

$$\frac{dS_{s_{1}}}{dt} = \rho_{1}V_{1} - \mu S_{s_{1}} = 0 \quad (24)$$

$$\frac{dS_{s_{2}}}{dt} = \rho_{2}V_{2} - \mu S_{s_{2}} = 0 \quad (25)$$

$$\frac{dS_{s_{3}}}{dt} = \rho_{3}V_{3} - \mu S_{s_{3}} = 0 \quad (26)$$

The solution for the disease-free equilibrium is derived as follows:

$$S_{P}^{*} = \frac{\mu}{\nu + \mu} \qquad (27)$$

$$V_{1}^{*} = \frac{\nu}{\omega + \rho_{1} + \mu} \cdot \frac{\mu}{\mu + \nu} \qquad (28)$$

$$V_{2}^{*} = \frac{\omega}{\omega_{1} + \rho_{2} + \mu} \cdot \frac{\nu}{\omega + \rho_{1} + \mu} \cdot \frac{\mu}{\mu + \nu} \qquad (29)$$

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$$V_{3}^{*} = \frac{\omega_{1}}{(\rho_{3} + \mu)} \cdot \frac{\omega}{(\omega_{1} + \rho_{2} + \mu)} \cdot \frac{\nu}{\omega + \rho_{1} + \mu} \cdot \frac{\mu}{\mu + \nu}$$
(30)

$$S_{S_{1}}^{*} = \frac{\rho_{1}}{\mu} V_{1}^{*} = \frac{\rho_{1}}{\omega + \rho_{1} + \mu} \cdot \frac{\nu}{\mu + \nu}$$
(31)

$$\mathbf{S}_{\mathbf{S}_{2}}^{*} = \frac{\rho_{2}}{\omega_{1} + \rho_{2} + \mu} \cdot \frac{\omega}{\omega + \rho_{1} + \mu} \cdot \frac{\nu}{\mu + \nu}$$
(32)

$$\mathbf{S}_{S_3}^* = \frac{\rho_3}{\left(\rho_3 + \mu\right)} \cdot \frac{\omega_1}{\left(\omega_1 + \rho_2 + \mu\right)} \cdot \frac{\omega}{\omega + \rho_1 + \mu} \cdot \frac{\nu}{\mu + \nu}$$
(33)

Accordingly, the reproduction number for the transmission dynamics system is governed by the formula (34):

$$\Re = \frac{\beta}{\gamma + \mu} \left[S_{p} + \varepsilon \mathbf{S}_{s} + \varepsilon_{v_{1}} V_{1} + \varepsilon_{v_{2}} V_{2} + \varepsilon_{v_{3}} V_{3} + \varepsilon_{1} S_{s_{1}} + \varepsilon_{2} S_{s_{2}} + \varepsilon_{3} S_{s_{3}} \right]$$

$$= \frac{\beta}{\gamma + \mu} \left\{ \frac{\mu}{v + \mu} \left[\frac{1 + \frac{v \varepsilon_{v_{1}}}{\omega + \rho_{1} + \mu} + \frac{v \varepsilon_{v_{2}}}{\omega + \rho_{1} + \mu} \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{1}{\omega} + \frac{1}{\rho_{3} + \mu} + \frac{1}{\rho_{3} + \rho_{3} + \mu} + \frac{\rho_{3} + \rho_{3} + \mu}{\omega_{1} + \rho_{2} + \mu} + \frac{\rho_{3} + \rho_{3} + \mu}{\omega_{1} + \rho_{2} + \mu} + \frac{\rho_{3} + \rho_{3} + \mu}{\rho_{3} + \mu} + \frac{1}{\rho_{3} + \rho_{3} + \mu} + \frac{1}{\rho_{3} + \mu} + \frac{1}{\rho_{3} + \rho_{3} + \mu} + \frac{1}{\rho_{3} + \mu} + \frac{1}{\rho_{3} + \rho_{3} + \mu} + \frac{1}{\rho_{3} + \rho_{3} + \mu} + \frac{1}{\rho_{3} +$$

The partial first-order differential concerning the administration rate of dose 1 is :

$$\frac{\partial \Re}{\partial \nu} = \frac{\beta}{\gamma + \mu} \begin{cases} \frac{-\mu}{\left(\nu + \mu\right)^{2}} \begin{bmatrix} 1 + \frac{\nu \varepsilon_{\nu_{1}}}{\omega + \rho_{1} + \mu} + \frac{\nu \varepsilon_{\nu_{2}}}{\omega + \rho_{1} + \mu} \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{\omega}{\omega_{1} + \rho_{2} + \mu} \end{bmatrix} + \\ \frac{\beta}{\left(\nu + \mu\right)^{2}} \begin{bmatrix} \frac{\rho_{1}\varepsilon_{1}}{\omega + \rho_{1} + \mu} + \frac{\rho_{2}\varepsilon_{2}}{\omega + \rho_{1} + \mu} \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{\omega}{\omega_{1} + \rho_{2} + \mu} + \frac{\omega}{\omega_{1} + \rho_{2} + \mu} \end{bmatrix} \end{cases}$$
(35)

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The sign of $\frac{\partial R}{\partial v}$ is indeterministic and hinges on multiple factors including dose administration rate, dose immunity waning rate, infection rate after vaccination, birth rate, and death rate. For reproduction number to be a monotonic decreasing function of the vaccination rate of dose 1

(i.e.,
$$\frac{\partial R}{\partial v} < 0$$
), the condition of below needs to be met:
 $(\omega_1 + \rho_2 + \mu)(\rho_3 + \mu)(\rho_1 \varepsilon_1 - v \varepsilon_{v_1}) + (\rho_3 + \mu)\omega(\rho_2 \varepsilon_2 - v \varepsilon_{v_2}) + \omega \omega_1(\rho_3 \varepsilon_3 - v \varepsilon_{v_3}) < (\omega + \rho_1 + \mu)(\omega_1 + \rho_2 + \mu)(\rho_3 + \mu)$ (36)

After reorganizing and simplification, we obtain the condition as stated in formula (37):

$$\frac{(\rho_{1}\varepsilon_{1}-\nu\varepsilon_{V_{1}})}{(\omega+\rho_{1}+\mu)} + \frac{\omega(\rho_{2}\varepsilon_{2}-\nu\varepsilon_{V_{2}})}{(\omega+\rho_{1}+\mu)(\omega_{1}+\rho_{2}+\mu)} + \frac{\omega\omega_{1}(\rho_{3}\varepsilon_{3}-\nu\varepsilon_{V_{3}})}{(\omega+\rho_{1}+\mu)(\omega_{1}+\rho_{2}+\mu)(\rho_{3}+\mu)} < 1$$
(37)

Following the similar vein as dose 1, we can derive the relative relation over the administration rate of dose 2:

$$\frac{\partial \Re}{\partial \omega} = \frac{\beta}{\gamma + \mu} \begin{cases} \frac{\mu}{\nu + \mu} \left[\frac{-\nu \varepsilon_{\nu_{1}}}{(\omega + \rho_{1} + \mu)^{2}} + \frac{\nu \varepsilon_{\nu_{2}}}{(\omega + \rho_{1} + \mu)^{2}} \frac{\rho_{1} + \mu}{\omega_{1} + \rho_{2} + \mu} + \frac{\rho_{1} + \mu}{\rho_{3} + \mu} + \frac{\rho_{2} + \mu}{\rho_{3} + \mu} + \frac{\rho_{2} + \mu}{\rho_{3} + \mu} + \frac{\rho_{2} + \mu}{\rho_{3} + \rho_{2} + \mu} + \frac{\rho_{2} + \mu}{\rho_{3} + \rho_{2} + \mu} + \frac{\rho_{3} + \mu}{\rho_{3} + \mu}$$

For R to be a monotonic decreasing function of dose 2 administration rate, it needs to satisfy the condition $\frac{\partial R}{\partial \omega} < 0$:

$$(\mu\varepsilon_{V_2} + \rho_2\varepsilon_2)(\rho_1 + \mu)(\rho_3 + \mu) + (\mu\varepsilon_{V_3} + \rho_3\varepsilon_3)(\rho_1 + \mu)\omega_1 < (\mu\varepsilon_{V_1} + \rho_1\varepsilon_1)(\omega_1 + \rho_2 + \mu)(\rho_3 + \mu)$$
(39)

And after simplification, we obtain the condition of (40):

$$\frac{\mu\varepsilon_{V_2} + \rho_2\varepsilon_2}{(\rho_2 + \mu)(\omega_1 + \rho_2 + \mu)} + \frac{(\mu\varepsilon_{V_3} + \rho_3\varepsilon_3)\omega_1}{(\rho_3 + \mu)(\rho_2 + \mu)(\omega_1 + \rho_2 + \mu)} < \frac{\mu\varepsilon_{V_1} + \rho_1\varepsilon_1}{(\rho_1 + \mu)(\rho_2 + \mu)}$$
(40)

Generalize the analysis to dose 3:

$$\frac{\partial \Re}{\partial \omega_{1}} = \frac{\beta}{\gamma + \mu} \begin{cases} \frac{\mu}{\nu + \mu} \left[\frac{\nu \varepsilon_{\nu_{2}}}{\omega + \rho_{1} + \mu} \frac{-\omega}{(\omega_{1} + \rho_{2} + \mu)^{2}} + \frac{\nu}{(\omega_{1} + \rho_{2} + \mu)^{2}} + \frac{\nu}{\rho_{3} + \mu} \right] + \\ \frac{\nu}{\nu + \mu} \left[\frac{\rho_{2} \varepsilon_{2}}{\omega + \rho_{1} + \mu} \frac{-\omega}{(\omega_{1} + \rho_{2} + \mu)^{2}} + \frac{\nu}{(\omega_{1} + \rho_{2} + \mu)^{2}} + \frac{\rho_{3} \varepsilon_{3}}{\omega + \rho_{1} + \mu} \frac{\omega}{(\omega_{1} + \rho_{2} + \mu)^{2}} + \frac{\rho_{3} \varepsilon_{3}}{\rho_{3} + \mu} \right] \end{cases}$$
(41)

For R to be a decreasing function of administration rate, i.e., $\frac{\partial u}{\partial \omega_1} < 0$, the given condition depicted in (42) needs to be met:



Name	Description	Baseline Value	References
α	relative infectiousness of I_s	1	[17][45][46]
β	transmission rate $R^*\gamma$		[17]
$\alpha_{_V}$	relative infectiousness of I_V	relative infectiousness of I_V 1	
α_1	relative infectiousness of I_{S_1} 1		Assumed
α_2	relative infectiousness of I_{s_2}	1	Assumed
α_{3}	relative infectiousness of I_{S_3}	1	Assumed
\mathcal{E}_1	susceptibility after the waning of dose 1 immunity	0.5	Assumed
\mathcal{E}_2	susceptibility after the waning of dose 2 immunity	0.5	Assumed
E ₃	susceptibility after the waning of dose 3 immunity	0.1	Assumed
\mathcal{E}_{V_1}	infection rate after vaccination with dose 1	0.1	Assumed
\mathcal{E}_{V_2}	infection rate after vaccination with dose 2	0.05	Assumed
\mathcal{E}_{V_3}	infection rate after vaccination with dose 3	0.05	Assumed
$\frac{1}{\rho_1}$	the inverse of dose 1 immunity waning rate	6.5 weeks	Assumed
$\frac{1}{ ho_2}$	the inverse of dose 2 immunity waning rate	{26,52} [weeks] Varies in the model.	Assumed
$\frac{1}{\rho_3}$	the inverse of dose 3 immunity waning rate	26 weeks	Assumed
μ	birth rate or death rate of population	0.02 per week	[17][45][46]
γ	the recovery rate of COVID-19 positive patients	1.4	[17][45][46]
V	vaccination rate of dose 1	0.01 per week	[17][45][46]
ω	vaccination rate of dose 2	0.05 per week	Assumed
ω_{1}	vaccination rate of dose 3{0,0.02,0.05} [pervaccination rate of dose 3week].Varies in the model.		Assumed
δ	waning rate to secondary susceptibility	1/3	Assumed
$\{S_{vax_1}, S_{vax_2}, S_{vax_3}, S_{vax_3}\}$	initiation of dose 1&2&3	{45,53,77}[week] {45,48,72} {45,48,80} {45,48,96} {47,50,74} Varies in the model.	Assumed
\mathbf{S}_{P}	initial size of the full susceptible population	1- <i>I</i> ₀	[17]
N	size of population	1	[17][45][46]
I_0	initial size of infection	1e-9	[17]
R	reproduction number	2.3	[17]
С	partially susceptible individuals vaccinated and immunity equivalent to dose 1	0.01	Assumed
d	partially susceptible individuals vaccinated and immunity equivalent to dose 1	0.01	Assumed

Name	Description	Baseline Value	References	
ν	vaccination rate of dose 1	0.01 per week	Baseline three-dose strategy	
ω	vaccination rate of dose 2	0.05 per week		
ω_1	vaccination rate of dose 3	0.02 [per week].		
ν	vaccination rate of dose 1	0.013 per week	Efficiency-enhanced primary	
ω	vaccination rate of dose 2	0.05*1.3 per week	vaccination case 1: rate of dose	
ω_1	vaccination rate of dose 3	0 [per week].	1&2 increases by 30%	
V	vaccination rate of dose 1	$v \cdot 1.5^{\frac{t}{t+1}}$	Efficiency-enhanced primary vaccination case 2: rate of dose	
ω	vaccination rate of dose 2	0.05 per week	1 increases over time	
ω_{1}	vaccination rate of dose 3	0 [per week].		
V	vaccination rate of dose 1	0.02per week	Efficiency-enhanced primary	
ω	vaccination rate of dose 2	0.05 per week	vaccination case 3: rate of dose	
ω_1	vaccination rate of dose 3	0 [per week].	1 doubles	

Table S2Simulation parameters for Fig. 3

Name	(A) Severe Variant	(B)	(C)	(D)
β	3.4	3.4	3.4	3.4
\mathcal{E}_1	0.6	0.5	0.4	0.4
\mathcal{E}_2	0.5	0.2	0.2	0.2
E ₃	0.5	0.1	0.1	0.1
\mathcal{E}_{V_1}	0.8	0.4	0.3	0.2
\mathcal{E}_{V_2}	0.7	0.1	0.05	0.03
\mathcal{E}_{V_3}	0.6	0.08	0.05	0.02
$\frac{1}{\rho_1}$	{5.2,13,26} weeks	{5.2,13,26} weeks	{5.2,13,26} weeks	{5.2,13,26} weeks
$\frac{1}{ ho_2}$	26 weeks	26 weeks	52 weeks	78 weeks
$\frac{1}{ ho_3}$	26 weeks	26 weeks	52 weeks	78 weeks
ω_{1}	5e(-7)(Close to boundary).	5e(-7)(Close to boundary).	5e(-7)(Close to boundary).	5e(-7)(Close to boundary).

Table S3	Simulation	parameters	for	Fig. 4	1
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Name	(A)	(B)	(C)
β	3.4	3.4	3.4
\mathcal{E}_1	0.5	0.5	0.5
\mathcal{E}_2	0.3	0.3	0.3
E3	0.1	0.08	0.05
\mathcal{E}_{V_1}	0.1	0.1	0.1
\mathcal{E}_{V_2}	0.05	0.05	0.05
\mathcal{E}_{V_3}	0.03	0.02	0.01
$\frac{1}{ ho_1}$	26 weeks	26 weeks	26 weeks
$\frac{1}{ ho_2}$	{13,26,52} weeks	{13,26,52} weeks	{13,26,52} weeks
$\frac{1}{ ho_3}$	52 weeks	52 weeks	52 weeks
V	0.01 per week	0.015 per week	0.02 per week

Table S5Simulation parameters for Fig. 5

Name	Description	Value	References
$\lambda_{_P}$	Likelihood of being severe infection at I_P	0.14	[17][45][46]
λ_{s}	Likelihood of being severe infection at $I_{\rm S}$	0.07	[17][45][46]
$\lambda_{_V}$	Likelihood of being severe infection at $I_{\scriptscriptstyle V}$	0.1	[17][45][46]
λ_1	Likelihood of being severe infection at I_V	0.1	[17][45][46]
λ_2	Likelihood of being severe infection at $I_{\scriptscriptstyle V}$	0.1	[17][45][46]
λ_3	Likelihood of being severe infection at $I_{\scriptscriptstyle V}$	0.1	Assumed

Table S5Simulation parameters for Fig. S2 and Fig. S3